

# A REVIEW OF

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A REVIEW OF

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## PREFACE

IN most high schools the course is finished by the end of the second year. In most students have forgotten many details, a thorough review is necessary in order to prepare for the entrance examinations at the beginning of the freshman year in college. Recent statistics show that many high schools are devoting at least two per cent of the senior year to a review of algebra.

For such a review the regular text is an embarrassment of riches the teacher

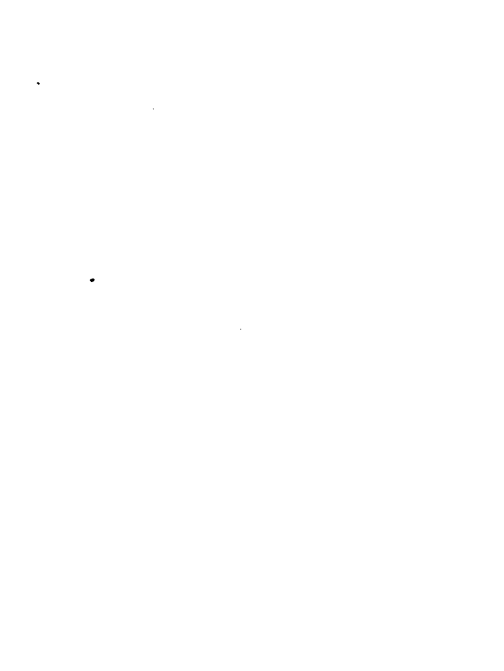
The definitions given in the book are reviewed as occasion arises, and the material can be profitably employed in any part of the Outline that the teacher desires, the example, or the formula.

The whole scheme of the book, with its set of problems represent a day's work, and can be applied to the Outlines or the book as a whole, covered more rapidly. By the omissions indicated in the book, the algebra can be readily covered in thirty-two lessons, thus less than eighteen weeks, of two periods each.

If a brief course is desired, the book (pp. 31 to 35, 50 to 52), may be used, the book, and the College Algebra may be omitted without marring the usefulness of the review.

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# OUTLINE OF ELEMENTARY INTERMEDIATE ALGEBRA

{ Factors; coefficient; exponent;  
term; algebraic sum; similar terms;  
homogeneous expression; linear equation;  
quadratic equation; root of an expression;  
conditional equation; prime quantity;  
lowest common factor (H. C. F.); lowest common multiple (L. C. M.); involution; evolution;  
rational number; real number; rational; surd;  
binomial surd; pure quadratic equation;  
quadratic equation; equation in two variables;  
form; simultaneous linear equations;  
simultaneous quadratic equations; discriminant



Special  
Rules for  
Multiplication  
and Division  
(continued)

5. Product of two binomials whose  
like terms are similar.  
 $(3x + 2)(4x + 3)$
6. Square of a polynomial.  
 $(m - n)^2$
7. Sum of two cubes.  
 $x^3 + y^3$
8. Difference of two cubes.  
 $\frac{x^3 - y^3}{x - y}$
9. Sum or difference of two fractions.  
 $\frac{x^7 + y^7}{x + y}, \frac{x^5 - y^5}{x - y}$
1. Common monomial factor.  
 $mx + my - mz$
2. Trinomial that is a perfect square.  
 $x^2 \pm 2ax + a^2$

Cases in  
Factoring  
(continued)

6. Sum or difference of  
 $\left\{ \begin{array}{l} \text{two cubes. See "} \\ \text{two like powers.} \end{array} \right.$

7. Common polynomial :

$$t^2p + t^2q - 2mp - 2mq \\ = t^2(p + q) - 2m(p + q)$$

8. Factor Theorem.

$$x^3 + 17x - 1$$

H. C. F.  
and  
L. C. M.

$$a^2 + 2a - 3 = (a + 3)(a - 1)$$

$$a^2 + 7a + 12 = (a + 3)(a + 4)$$

$$a^4 + 27a = a(a + 3)(a^2 - 3a + 9)$$

$$\text{H. C. F.} = a + 3.$$

$$\text{L. C. M.} = (a + 3)(a - 1)(a + 4)$$

Reduction to lowest terms

Reduction of a mixed  
fraction.

Reduction of an improper  
number.

Fractions

Addition and subtraction

## Evolution

Law of signs.

Evolution of m

Square root of a

Square root of a

Optional { Cub  
Cub

## Theory of Exponents

Proofs:  $a^m \cdot a^n$

$$\sqrt[n]{a^{mn}}$$

Meaning of { f  
z  
n

T

T

Radicals  
(continued)

Multiplication and division of radicals

Rationalization { Monomials  
Binomials  
Trinomials

Square root of a binomial

Radical equations. Also extraneous roots.

{ Pure.  $x^2 = a$ .

{ Affected.  $ax^2 + bx + c = 0$

Methods of solving { Completing the square  
Factoring  
Formula

Equations in the quadratic form

**Simultaneous  
Quadratics**

CASE I.

{ One e  
The o

CASE II.

{ Both  
the

CASE III.

{ Any  
 $x^2 -$   
 $x^2 \pm$

CASE IV.

{ Both  
me  
onc  
oth

Ratio and  
Proportion

Proportionals { mean,  
third,  
fourth.

- Theorems { 1. Product of  
of extrem  
2. If the pro  
equals th  
numbers,  
3. Alternation  
4. Inversion.  
5. Composition  
6. Division.  
7. Composition  
8. In a series o  
of the an  
of the co  
cedent, et

Special method of proving  
portion. Let  $\frac{a}{b} = x$ ,  $a =$

(Development of formulas

# A REVIEW OF

## ORDER OF OPERATIONS, EVALUATION

Order of operations :

First of all, raising to a power and

Next, multiplication and division

Last of all, addition and subtraction

Find the value of :

1.  $5 \cdot 2^2 - \sqrt{25} \div 5 + 2^2 \cdot 8 \div 4 -$

2.  $\frac{3 \times 6 \div 9}{2} - 2\sqrt{100} \div 5 + 4 \cdot 2$

12/14/22 3.  $9 \cdot 2 \div 6 + 3 - 2 \cdot 4^2 \div \sqrt[3]{8} - 4$

Evaluate :

4.  $\frac{a^4 - a^3 + b^3}{\sqrt{a^2b^2}} + \frac{c\sqrt{a} + a^3bc}{abc}$ , if  $a$

# SPECIAL RULES OF MULTIPLI

Give results by inspection :

1.  $(g + \frac{1}{2}k)^2$ . 9
2.  $\left(s - \frac{2m}{3}\right)^2$ . 10
3.  $(2v + 3w)(2v - 3w)$ .
4.  $(x + 3ts)(x - 7ts)$ . 11
5.  $(2l + 3g)(4l - 11g)$ . 12
6.  $\left(a - \frac{2b}{3} + c - d\right)^2$ . 13
7.  $\frac{x^3 + 8m^3}{x + 2m}$ . 14
8.  $\frac{y^3 - 27k^{3m}}{y - 3k^m}$ . 15
16.  $(k^{32} + 1)(k^{16} + 1)(k^8 + 1)(k^4 + 1)$



## CASES IN

The number of terms in  
 clue to the possible cases under  
 ing the *test* for each and elimi-  
 one, the right case is readily  
 terms in the expression and  
 of the Cases in Factoring are  
 vitally important part of alge-

CASE I. A common monomial  
 ber of terms.

$$5cx -$$

CASE II. A trinomial that  
 terms.

$$x^2 \pm$$

CASE III. The difference of

$$\begin{aligned}
 c^4 + c^2d^2 + d^4 &= c^4 + c^2d^2 + d^4 \\
 &= (c^2 + d^2)^2 - c^2d^2 \\
 &= (c^2 + d^2 + cd)(c^2 + d^2 - cd)
 \end{aligned}$$

CASE IV. A trinomial of the form

$$x^2 + x - 30 =$$

CASE V. A trinomial of the form  
terms.

$$20x^2 + 7x -$$

CASE VI. A. The sum or difference  
terms.

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

B. The sum or difference  
Two terms.

Review the *Cases* (pages) and write out the

1.  $8a^{13} + am^{12}$ .

2.  $x^7 + y^7$ .

3.  $4x^2 + 11x - 3$ .

4.  $m^2 + n^2 - (1 + 2mn)$ .

5.  $-x^2 + 2x - 1 + x^4$ .

6.  $x^{16} - y^{16}$ . (Five factors)

7.  $(x + 1)^2 - 5x - 29$ .

8.  $x^4 + x^2y^2 + y^4$ .

9.  $x^4 - 11x^2 + 1$ .

10.  $x^{2m} + 2 + \frac{1}{x^{2m}}$ .

21.  $gt - gk + gl^2$

22.  $(m - n)(2a^2$

## HIGHEST COMMON FACTOR AND MULTIPLE

Define H.C.F. and L.C.M.

Find by factoring the H.C.F. and

1.	$3x^2 - 3x,$	5.
	$12x^2(x^2 - 1),$	
	$18x^3(x^3 - 1).$	

2.	$(x^2 - 1)(x^2 + 5x + 6),$	6.
	$(x^2 + 3x)(x^2 - x - 6).$	
	<i>(Harvard.)</i>	

3.	$x^2 - y^2,$	
	$x^2 + y^2,$	7.
	$x^3 + y^3,$	
	$x^6 + y^6,$	
	$x^6 - y^6.$	
	<i>(College Entrance Board.)</i>	

4.	$x^3 + x^2 - 2,$	8.
----	------------------	----

## FRACTIONS

Define: fraction, terms of a fraction.

Look up *the law of signs* as it applies to fractions; for this, fractions in algebra are the same as they are in arithmetic.

1. Reduce to lowest terms:

$$(a) \frac{32}{24}; \quad (b) \frac{a^6 - x^6}{a^4 - x^4}; \quad (c) \frac{(a + b)^2}{(a + b)}$$

2. Reduce to a mixed expression:

3. Reduce to an improper fraction:

$$(a) 45\frac{1}{8}; \quad (b) 9\frac{1}{2} \text{ qt.}; \quad (c) \dots$$

Add:

$$4. \frac{5}{18} + \frac{7}{9} + \frac{11}{16} + \frac{5}{8}. \qquad 5. \dots$$

$$6. \frac{1}{x(x-a)(x-b)} + \frac{1}{a(a-x)(x-b)}$$

Multiply:

$$7. \frac{72}{121} \times \frac{55}{56} \times \frac{77}{90}.$$



# FRACTION

1. Solve for each letter in

2. Solve and check :

$$\frac{5x+2}{3} - \left( 3 - \frac{3x-1}{2} \right)$$

3. Solve and check:

$$\frac{1}{2} \left( x - \frac{a}{3} \right) - \frac{1}{3} \left( x - \frac{a}{4} \right) +$$

4. Solve (after looking up

$$\frac{3x-1}{30} + \frac{4x-7}{15} - \frac{x}{4}$$

5. Solve by the special *sho*

$$\frac{1}{x-2} - \frac{1}{x-3} = \frac{1}{x-4}$$

6. At what time between  
watch (a) opposite each other  
gether?

Work out (a) and state the

## SIMULTANEOUS EQUATIONS

NOTE. Up to this point each topic presented has extended the preceding topics. For example, factoring, rules of multiplication and division; H. C. F. and L. C. M.; addition and subtraction of fractions and fractions; H. C. F. and L. C. M., etc. From this point on, however, the connection is not so marked, and miscellaneous examples already covered will be given very frequently in order that the subject be fresh in mind.

1. Solve by three methods — addition and subtraction, and comparison : 
$$\begin{cases} 5x + y = 11, \\ 3x + 2y = 1. \end{cases}$$

Solve and check:

$$2. \begin{cases} 12R_1 - 11R_2 = b + 12c, \\ R_1 + R_2 = 2b + c. \end{cases} \quad 3. \begin{cases} \frac{r-s}{2} = \dots \\ \frac{r+s}{2} = \dots \end{cases}$$

4. One half of A's marbles exceeds one half of B's together by 2; twice B's marbles falls short of A's together by 16; if C had four more marbles than B, he would have as many as A.



# SIMULTANEOUS EQUATIONS

1. Solve  $\begin{cases} \frac{3}{4x} - \frac{5}{3y} = 11\frac{1}{2}, \\ \frac{5}{8x} - \frac{3}{2y} = 10\frac{1}{4}. \end{cases}$  ing  
the  
clear

2. Solve  $\begin{cases} \frac{1}{x} - \frac{1}{y} - \frac{1}{z} = 1 \\ \frac{1}{y} - \frac{1}{z} - \frac{1}{x} = 1 \\ \frac{1}{z} - \frac{1}{x} - \frac{1}{y} = 1 \end{cases}$

3. Solve graphically and algebraically

4. Solve graphically and algebraically

Review :

5. The squares of the numbers

6. The cubes of the numbers fr

7. The fourth powers of the nu

## SQUARE R

Find the square root of:

1.  $1 + 16 m^6 - 40 m^4 + 10 m^2$

2.  $\frac{a^2}{x^2} + \frac{6 a}{x} + 11$

3. Find the square root to three

4. Find the square root of 337,

5. Find the square root of 1823

6. Find to four decimal places

7. Add  $\frac{2}{(x-1)^3} + \frac{1}{(1-x)^2}$

8. Find the value of:

$$\frac{\sqrt[3]{64} \cdot 12}{24} \div 2 \times 3 - \frac{2 \cdot 7^2}{14} \div 7$$

9. Simplify  $[(x+y)^5 + (x-y)^5]$

10. Solve by the short method:

## THEORY OF

Review the proofs, for positive

$$\text{I. } a^m \times a^n = a^{m+n}.$$

$$\text{II. } \frac{a^m}{a^n} = a^{m-n}.$$

$$\text{III. } (a^m)^n = a^{mn}.$$

To find the meaning of a fract

Assume that Law I holds fo

If so,  $a^{\frac{2}{3}} \cdot a^{\frac{2}{3}} \cdot a^{\frac{2}{3}}$

Hence,  $a^{\frac{2}{3}}$  is *one of the three equal factors*  
of  $a^2$ .  $\therefore a^{\frac{2}{3}}$

In the same way,  $a^{\frac{4}{5}} \cdot a^{\frac{4}{5}} \cdot a^{\frac{4}{5}} \cdot a^{\frac{4}{5}} \cdot a^{\frac{4}{5}}$

Hence,  $a^{\frac{4}{5}}$  is *one of the five equal factors*  
of  $a^4$ .  $\therefore a^{\frac{4}{5}}$

In the same way, in general,  $a^{\frac{m}{n}}$

Hence, *the numerator of a*

## THEORY OF EXPONENTS

Rules :

*To multiply quantities having the same*

*To divide quantities having the same base*

*To raise a quantity to a power, multiply*

*To extract a root, divide the exponent of  
of the root.*

1. Find the value of  $3^2 - 5 \times 4^0 +$

2. Find the value of  $8^{-\frac{2}{3}} + 9^{\frac{3}{2}} - 2^{-}$

Give the value of each of the following

3.  $\frac{3^0}{5}, \frac{3}{5^0}, \frac{3^0}{5^0}, 3^0 \times 5, 3 \times 5^0, 3^0 \times 5^0$

4. Express  $7^0$  as some power of 7 divided by

Simplify :

5.  $16^{\frac{1}{3}} \cdot 2^{\frac{1}{2}} \cdot 32^{\frac{5}{6}}$  (Change to

## THEORY OF EXPONENTIALS

Solve for  $x$ :

1.  $x^{\frac{2}{3}} = 4.$

Factor:

3.  $x^{\frac{2}{3}} - 9.$

4.  $x^{\frac{3}{5}} + 27.$

7. Find the H. C. F. and L. C. M. of

$$a^2 + a^{\frac{3}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{3}{2}} - b^2,$$

8. Simplify the product of

$$(axy^{-1})^{\frac{1}{2}}, (bxy^{-2})^{\frac{1}{3}}, \text{ and } (cxy^{-3})^{\frac{1}{4}}.$$

9. Find the square root of:

$$25 a^{\frac{4}{3}}b^{-3} - 10 a^{\frac{2}{3}}b^{-\frac{3}{2}} - 4 b^{-3}.$$

10. Simplify  $\sqrt[5]{\frac{2^{n+2}}{4^{-n}} \div \frac{8^n}{9^3}}.$

## RADICALS

1. Review all definitions in Radicals, transforming and simplifying radicals.  
*its simplest form?*

2. Simplify (to simplest form):  $\sqrt[2]{\frac{2}{3}}$   
 $\frac{2a}{b}\sqrt{\frac{8b^2}{27a}}; \sqrt[2n]{\frac{5}{x^n}}; (a+b)^2\sqrt[3]{\frac{-a^4}{(a+b)^5}}; \sqrt{2}$

3. Reduce to entire surds:  $2\sqrt{3};$   
 $-3\sqrt[3]{2}; 3a\sqrt[3]{\frac{a+2}{6a^2}}; (a+2y)\sqrt{\frac{a-2y}{a+2y}}$

4. Reduce to radicals of lower order

$$\sqrt[4]{a^2}; \sqrt[6]{a^3}; \sqrt[6]{27a^3}; \sqrt[12]{81a^4x^8}$$

5. Reduce to radicals of the same degree  
 $\sqrt{7}$  and  $\sqrt[3]{11}; \sqrt[3]{5}$  and  $\sqrt[4]{3}; \sqrt[6]{7}$  and  
 $\sqrt[x]{c^y}, \sqrt[y]{c^z},$  and  $\sqrt[z]{c^x}.$

6. Which is greater,  $\sqrt{3}$  or  $\sqrt[3]{4}$ ?  $\sqrt[3]{5}$

7. Which is greatest,  $\sqrt{2}, \sqrt[3]{5},$  or  $\sqrt[4]{16}$ ?

## RADICALS

The most important principle is

$$(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}. \quad \text{Hence } \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

From this also 
$$\frac{\sqrt[n]{ab}}{\sqrt[n]{a}} = \sqrt[n]{b}$$

Multiply :

1.  $2\sqrt[3]{4}$  by  $3\sqrt[3]{6}$ .

2.  $\sqrt{2}$  by  $\sqrt[3]{3}$ .

5.  $\sqrt{2} + \sqrt{3} - \sqrt{5}$  by  $\sqrt{2} - \sqrt{3}$

6.  $-\frac{p}{2} + \frac{\sqrt{p^2 - 4q}}{2}$  by  $-\frac{p}{2} - \frac{\sqrt{p^2 - 4q}}{2}$

Divide :

7.  $\sqrt{27}$  by  $\sqrt{3}$ .

8.  $4\sqrt{18}$  by  $5\sqrt{32}$ .

11.  $6\sqrt{105} + 18\sqrt{40} - 45\sqrt{12}$

12.  $10\sqrt[3]{18} - 4\sqrt[3]{60} + 5\sqrt[3]{100}$

# MISCELLANEOUS EXAMPLES, TO QUADRATICS

Results by inspection, examples 1-10.

Divide:

Multiply:

$$1. \frac{x^{\frac{5}{17}} + y^{\frac{5}{17}}}{x^{\frac{1}{17}} + y^{\frac{1}{17}}}.$$

$$5. \left( a^{-\frac{3}{4}} \right).$$

$$2. \frac{x - y}{x^{\frac{1}{3}} - y^{\frac{1}{3}}}.$$

$$6. (K^{-\frac{2}{7}})$$

$$7. (r^{2s} +$$

$$3. \frac{m^2 + n^2}{m^{\frac{2}{3}} + n^{\frac{2}{3}}}.$$

$$8. \left( a^{-2} - \right.$$

$$4. \frac{x - y^2}{\sqrt[3]{x} - \sqrt[3]{y^2}}.$$

$$9. (3 K^x$$

$$10. (2 y^{\frac{2}{7}} -$$

Factor:

$$11. x^{\frac{2}{3}} - 64.$$

$$13. b^{\frac{3}{2}} - 8$$

$$12. y^{\frac{3}{5}} + 27.$$

$$14. 3 p -$$

Factor, using radicals instead of exponents.



3. Find the square root of  $8114.1061$ . What, then, is the square root of  $.0081141061$ ? of  $811410.61$ ? From any of the above can you determine the square root of  $.081141061$ ?

4. The H.C.F. of two expressions is  $a(u-b)$ , and their L.C.M. is  $a^2b(u+b)(u-b)$ . If one expression is  $ab(u^2-b^2)$ , what is the other?

5. Solve (short method):

$$\frac{5}{7-x} - \frac{2\frac{1}{4}x-3}{4} - \frac{x+11}{8} + \frac{11x+5}{16} = 0.$$

6. Solve

$$\begin{cases} \frac{2}{m} - \frac{3}{n} + \frac{10}{p} = -3, \\ \frac{4}{m} + \frac{5}{p} + \frac{6}{n} = 15, \\ \frac{1}{m} - \frac{1}{n} + \frac{5}{p} = -\frac{1}{2}. \end{cases}$$

7. Simplify  $21\sqrt{\frac{1}{3}} - 5\sqrt{\frac{1}{3}} + 6\sqrt{\frac{1}{3}} - 10\sqrt{3\frac{1}{3}} + 4^0\sqrt{11\frac{1}{3}}$ .

8. Does  $\sqrt{16} \times 25 = 4 \times 5$ ? Does  $\sqrt{16} + 25 = 4 + 5$ ?

9. Write the fraction  $\frac{5}{4+2\sqrt{3}}$  with rational denominator,

and find its value correct to two decimal places.

10. Simplify 
$$\frac{\left\{ \sqrt{\frac{p+\sqrt{p^2-q}}{2}} + \sqrt{\frac{p-\sqrt{p^2-q}}{2}} \right\}^2}{p+\sqrt{q}}.$$

(Princeton.)

Simplify  $\frac{2^{n+1} - 2(2^n)}{2(2^{n+1})}$ . (Univ. of Penn.)

Find the value of  $\frac{1 + 8^{\frac{x}{3}}}{(8x)^{\frac{1}{3}} + 10^{x-2}}$ , when  $x = 2$ . (Cornell.)

Find the value of  $x$  if  $\begin{cases} x^a = y^4, \\ y^{\frac{a}{3}} = 9. \end{cases}$  (M. I. T.)

A fisherman told a yarn about a fish he had caught. If the fish were half as long as he said it was, it would be 10 inches more than twice as long as it is. If it were 4 inches longer than it is, and he had further exaggerated its length by adding 4 inches, it would be  $\frac{1}{3}$  as long as he now said it was. How long is the fish, and how long did he first say it was? (M. I. T.)

The force  $P$  necessary to lift a weight  $W$  by means of a machine is given by the formula

$$P = a + bW,$$

where  $a$  and  $b$  are constants depending on the amount of friction in the machine. If a force of 7 pounds will raise a weight of 10 pounds, and a force of 13 pounds will raise a weight of 50 pounds, what force is necessary to raise a weight of 40 pounds? (Harvard.)

Reduce to the simplest form:  $\sqrt[n]{\frac{4}{2^{n+2}}} \cdot \frac{ax(a^{-1}x - ax^{-1})}{x^{\frac{1}{2}} - a^{\frac{1}{2}}} \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}}a^{\frac{1}{2}} + a^{\frac{1}{2}}$

Determine the H.C.F. and L.C.M. of  $(xy - y^2)^3$  and  $x^2y$ . (College Entrance Board.)

$$\frac{\left(a^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}}\right)^2 - \left(\frac{1}{a^{\frac{1}{2}}} - x^{\frac{1}{2}}\right)^2}{x + \sqrt{a^2 + x^2}} \quad (M. I. T.)$$

3. Find  $\sqrt{7} - \sqrt{13}$ .

4. Expand  $(\sqrt{a^3} - \sqrt{b^6})^6$ .

5. Expand and simplify  $(1 - 2\sqrt{3} + 3\sqrt{2})^2$ .

6. Solve the simultaneous equations  $\begin{cases} x^{\frac{1}{2}} + 2y^{\frac{1}{2}} = 7, \\ 2x^{\frac{1}{2}} - y^{\frac{1}{2}} = 3. \end{cases}$  (Yale.)

7. Find to three places of decimals the value of

$$\sqrt{\frac{(a+b)^{\frac{1}{2}}}{(11a+b^2)^{\frac{1}{2}}} \cdot \frac{(a^3-b^3)^{\frac{1}{2}}}{(a-b)^{\frac{1}{2}}}},$$

when  $a=5$  and  $b=3$ . (Columbia.)

8. Show that  $\frac{10 - 4\sqrt{5}}{5 + 3\sqrt{5}}$  is the negative of the reciprocal of  $\frac{10 + 4\sqrt{5}}{5 - 3\sqrt{5}}$ . (Columbia.)

9. Solve and check  $\frac{5}{\sqrt{3x+2}} = \sqrt{3x+2} + \sqrt{3x-1}$ . (Yale.)

10. Assuming that when an apple falls from a tree the distance ( $S$  meters) through which it falls in any time ( $t$  seconds) is given by the formula  $S = \frac{1}{2}gt^2$  (where  $g = 9.8$ ), find to two decimal places the time taken by an apple in falling 15 meters. (College Entrance Board.)

# ARITHMETIC

$$p = br$$

$$i = prt$$

$$a = p + prt$$

## GEOMETRY

$$K = \frac{1}{2} bh$$

$$K = bh$$

$$K = \frac{a^2}{4} \sqrt{3}$$

$$K = \frac{1}{2} (b + b') h$$

$$K = \pi R^2$$

$$C = 2 \pi R$$

$$K = \pi RL$$

$$S = \frac{1}{2} \pi R^2$$

$$V = \pi R^2 H$$

$$V = \frac{1}{3} \pi R^2 H$$

$$V = \frac{4}{3} \pi R^3$$

$$S = \frac{\pi R^2 E}{180}$$

$$\frac{C}{C'} = \frac{R}{R'}$$

$$\frac{K}{K'} = \frac{R^2}{R'^2}$$

## PHYSICS

$$v = gt$$

$$s = \frac{1}{2} gt^2$$

$$s = \frac{v^2}{2g}$$

$$C = \frac{E}{R}$$

$$E = \frac{mv^2}{2g}$$

$$e = \frac{4 \pi^2 m}{bh^3 m}$$

$$E = \frac{mv^2}{2}$$

$$t = \pi \sqrt{\frac{l}{g}}$$

$$F = \frac{mV^2}{r}$$

$$mh = \frac{mv^2}{2g}$$

$$R = \frac{gs}{g + s}$$

$$E = \frac{4 \pi^2 m v}{g}$$

$$C = \frac{5}{9} (F - 32)$$

Review the first (or usual) method of completing the square. Solve by it the following:

3.  $x^2 + 10x = 24$ .

5.  $\frac{x-1}{2} + \frac{2}{x-1} = 2\frac{1}{2}$ .

4.  $2x^2 - 5x = 7$ .

6.  $ax^2 + bx + c = 0$ .

Review the solution by factoring. Solve by it the following:

7.  $x^2 + 8x + 7 = 0$ .

9.  $3 = 10x - 3x^2$ .

8.  $24x^2 = 2x + 15$ .

10.  $-7 = 6x - x^2$ .

Solve, by factoring, these equations, which are not quadratics:

11.  $x^4 = 16$ .

12.  $x^3 = 8$ .

13.  $x^3 = x$ .

Review the solution by formula. Solve by it the following:

14.  $5x^2 - 6x = 8$ .

15.  $\frac{1}{2}(x+1) - \frac{x}{3}(2x-1) = -12$ .

16.  $x^2 + 4ax = 12a^2$ .

17.  $3x^2 = 2rx + 2r^2$ .

Solve graphically:

18.  $x^2 - 2x - 8 = 0$ .

19.  $x^2 + x - 2 = 0$ .

**Reference:** The chapter on Quadratic Equations in any algebra (first part of the chapter).

$$x^4 - 5x^2 = -4.$$

$$3. \quad 2\sqrt[3]{x^2} - 3\sqrt[3]{x} = 2.$$

$$\frac{x+3}{x-3} + 6 = 5\sqrt{\frac{x+3}{x-3}}. \quad \left( \text{Let } y = \sqrt{\frac{x+3}{x-3}} \text{ and substitute.} \right)$$

$$3x^2 - 4x + 2\sqrt{3x^2 - 4x - 6} = 21.$$

$$x^2 + 5x - 5 = \frac{6}{x^2 + 5x}.$$

we and check:

$$\sqrt{x+7} + \sqrt{3x-2} = \frac{4x+9}{\sqrt{3x-2}}.$$

$$\sqrt{x^2-5} + \frac{6}{\sqrt{x^2-5}} = 5.$$

$$\frac{10w}{\sqrt{10w-9}} - \sqrt{10w+2} = \frac{2}{\sqrt{10w-9}}.$$

we results by inspection:

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}).$$

$$(\sqrt{10} + \sqrt{19})(\sqrt{10} - \sqrt{19}).$$

How many gallons each of cream containing 33 % fat and milk containing 6 % butter fat must be mixed to produce 10 gallons of cream containing 25 % butter fat?

I have \$6 in dimes, quarters, and half-dollars, there being 15 coins in all. The number of dimes and quarters together is twice the number of half-dollars. How many coins of each kind are there?

(College Entrance Board.)

Exercise: The last part of the chapter on Quadratic Equations in any algebra.

Or,

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0. \quad (2)$$

To derive the formula, we have by transposing

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

Completing the square,

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}.$$

Extracting square root,  $x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$

Transposing,  $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$

Hence,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

These two values of  $x$  we call *roots*.

For convenience represent them by  $r_1$  and  $r_2$ .

Hence,  $r_1 = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}.$

$$r_2 = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}.$$

Adding,

$$r_1 + r_2 = -\frac{2b}{2a} = -\frac{b}{a}. \quad (3)$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}. \quad (4)$$

hence we have shown that  $\begin{cases} r_1 + r_2 = -\frac{b}{a}, \\ \text{and } r_1 r_2 = \frac{c}{a}. \end{cases}$

, referring to equation (2) above, we have the following rule :  
*When the coefficient of  $x^2$  is unity, the sum of the roots is the coefficient of  $x$  with the sign changed; the product of the roots is the independent term.*

EXAMPLES :

$$\begin{aligned} x^2 - 9x + 21 &= 0. & \begin{cases} \text{Sum of the roots} = 9. \\ \text{Product of the roots} = 21. \end{cases} \\ 3x^2 - 7x - 18 &= 0. & \begin{cases} \text{Sum of the roots} = \frac{7}{3}. \\ \text{Product of the roots} = -6. \end{cases} \\ -21x = 17 - 4x^2. & & \begin{cases} \text{Sum of the roots} = \frac{21}{4}. \\ \text{Product of the roots} = -\frac{17}{4}. \end{cases} \end{aligned}$$

To find the nature or character of the roots.

As before,

$$r_1 = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a},$$

$$r_2 = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}.$$

As  $\sqrt{b^2 - 4ac}$  determines the nature or character of the roots; hence it is called the *discriminant*.



$$1. \quad x^2 - 4x + 2 = 0.$$

$\sqrt{b^2 - 4ac} = \sqrt{16 - 8} = \sqrt{8}$ .  $\therefore$  The roots are real, unequal, and irrational.

$$2. \quad x^2 - 4x + 6 = 0.$$

$\sqrt{b^2 - 4ac} = \sqrt{16 - 24} = \sqrt{-8}$ .  $\therefore$  The roots are imaginary and unequal.

$$3. \quad x^2 - 4x + 4 = 0.$$

$\sqrt{b^2 - 4ac} = \sqrt{16 - 16} = \sqrt{0}$ .  $\therefore$  The roots are real, equal, and rational.

III. To form the quadratic equation when the roots are given.

Suppose the roots are 3, -7.

$$\text{Then, } \begin{array}{l} x = 3, \\ x = -7. \end{array}$$

$$\text{Or, } \begin{array}{l} x - 3 = 0, \\ x + 7 = 0. \end{array}$$

Multiplying to get a quadratic,

$$\frac{(x-3)(x+7) = 0.}{(x-3)(x+7) = 0.}$$

$$\text{Or, } x^2 + 4x - 21 = 0.$$

Or, use the sum and product idea developed on the preceding page. The coefficient of  $x^2$  must be unity.

Add the roots and change the sign to get the coefficient of  $x$ .

Multiply the roots to get the independent term.

$\therefore$  The equation is  $x^2 + 4x - 21 = 0$ .

In the same way, if the roots are  $\frac{2+\sqrt{3}}{7}$ ,  $\frac{2-\sqrt{3}}{7}$ , the equation is

$$x^2 - \frac{4}{7}x + \frac{1}{49} = 0.$$

$$2. \quad 9x^2 - 6x + 1 = 0.$$

$$3. \quad x^2 + 2x + 9731 = 0.$$

$$4. \quad 16 + \frac{5}{x} = \frac{17}{x^2}.$$

$$x - 3$$

$$6. \quad (x + 7)(x - 6) = 70.$$

$$7. \quad x^2 - x\sqrt{2} = 3.$$

$$8. \quad pr^2 + qr + s = 0.$$

Form the equations whose roots are:

$$9. \quad 5, -3.$$

$$10. \quad \frac{2}{3}, \frac{5}{8}.$$

$$11. \quad c + d, c - d.$$

$$12. \quad -3, -5.$$

$$13. \quad \frac{2 \pm \sqrt{-3}}{5}.$$

$$14. \quad \frac{8}{3} + \frac{2}{3}\sqrt{37}, \frac{8}{3} - \frac{2}{3}\sqrt{37}.$$

$$15. \quad \frac{-2 \pm \sqrt{-2}}{2}.$$

13. Solve  $x^2 - 3x + 4 = 0$ . Check by substituting the roots of  $x$ ; then check by finding the sum and the product of roots. Compare the amount of labor required in each case.

14. Solve  $(x - 3)(x + 2)(x^2 + 3x - 4) = 0$ .

15. Is  $e^{1x} + 2e^{3x} + e^{2x} + 2e^x + 2 + e^{-2x}$  a perfect square?

16. Find the square root (short method):

$$(x^2 - 1)(x^2 - 3x + 2)(x^2 - x - 2).$$

$$17. \text{ Solve } \frac{1.2x - 1.5}{1.5} + \frac{.4x + 1}{.2x - .2} = \frac{.4x + 1}{.5}.$$

18. The glass of a mirror is 18 inches by 12 inches, and it has a frame of uniform width whose area is equal to that of the glass. Find the width of the frame.

Simultaneous  
Quadratics

METHOD: Solve for  $x$  in terms of  $y$ ,  
or *vice versa*, in the linear and sub-  
stitute in the quadratic.

CASE II. { Both equations homogeneous and  
of the second degree.

$$\begin{cases} x^2 - xy + y^2 = 39, \\ 2x^2 - 3xy + 2y^2 = 43. \end{cases}$$

METHOD: Let  $y = vx$ , and substitute  
in both equations.

ALTERNATE METHOD: Solve for  $x$  in  
terms of  $y$  in one equation and sub-  
stitute in the other.

CASE III. Any two of the  
quantities  $\left\{ \begin{array}{l} x + y \\ x^2 + y^2 \\ xy \\ x - y \\ x^3 + y^3 \\ x^3 - y^3 \\ x^2 + xy + y^2 \\ x^2 - xy + y^2 \end{array} \right\}$  given.

$$\begin{cases} x + y = 5, \\ x^2 - xy + y^2 = 7. \end{cases}$$

METHOD: Solve for  $x + y$  and  $x - y$ ;  
then add to get  $x$ , subtract to get  $y$ .

$$\begin{cases} x + y = 2. \end{cases}$$

METHOD: Let  $x = u + v$  and  $y = u - v$ , and substitute in both equations.

I. Consider some compound quantity

like  $xy$ ,  $\sqrt{x - y}$ ,  $\sqrt{xy}$ ,  $\frac{x}{y}$ , etc., as the unknown, at first. Solve for the compound unknown, and combine the resulting equation with the simpler original equation.

$$\begin{cases} x^2y^2 + xy = 6, \\ x + 2y = -5. \end{cases}$$

Special  
Devices

II. Divide the equations member by member. Then solve by Case I, II, or III.

$$\begin{cases} x^3 - y^3 = 152, \\ x - y = 2. \end{cases}$$

III. Eliminate the quadratic terms. Then solve by Case I, II, or III.

$$\begin{cases} xy + x = 15, \\ xy + y = 16. \end{cases}$$

ultaneous  
dratics  
continued)

$$2. \begin{cases} 2x + y = 1, \\ xy + y^2 = 14. \end{cases}$$

$$9. \begin{cases} x^2 + 2xy + y^2 = 3, \\ 3x^2 + 2xy - y^2 = 3. \end{cases}$$

$$3. \begin{cases} x^2 + y^2 = 25, \\ x + y = 1. \end{cases}$$

$$10. \begin{cases} x^5 + y^5 = 242, \\ x + y = 2. \end{cases}$$

$$4. \begin{cases} x^4 + y^4 = 2, \\ x - y = 2. \end{cases}$$

$$11. \begin{cases} x - y + \sqrt{x - y} = 6, \\ xy = 5. \end{cases}$$

$$5. \begin{cases} x^3 + y^3 = 28, \\ x + y = 4. \end{cases}$$

$$12. \begin{cases} 4x^2 - x + y = 67, \\ 3x^2 - 3y = 27. \end{cases}$$

$$6. \begin{cases} x^2y^2 + xy - 12 = 0, \\ x + y = 4. \end{cases}$$

$$13. \begin{cases} x - y - \sqrt{x - y} = 2, \\ x^3 - y^3 = 2044. \end{cases} \quad (\text{Yale.})$$

$$7. \begin{cases} 2xy - x + 2y = 16, \\ 3xy + 2x - 4y = 10. \end{cases}$$

$$14. \begin{cases} x^2 + xy + x = 14, \\ y^2 + xy + y = 28. \end{cases}$$

(Princeton.)

$$15. \begin{cases} x^2 + y^2 = 13, \\ y^2 = 4(x - 2). \end{cases} \quad \text{Plot the graph of each equation.}$$

(Cornell.)

$$16. \begin{cases} x^2 + y^2 = xy + 37, \\ x + y = xy - 17. \end{cases}$$

(Columbia.)

In grouping the answers, be sure to associate each value of  $x$  with the corresponding value of  $y$ .

17. The course of a yacht is 30 miles in length and is in the shape of a right triangle one arm of which is 2 miles longer than the other. What is the distance along each side?

**Reference:** The chapter on Simultaneous Quadratics in any algebra.

Find a third proportional to 4 and 7; 5 and 10;  $a^2 - 9$   
- 3.

Find a fourth proportional to 2, 5, and 4; 35, 20, and 14.

Write out the proofs for the following, stating the  
em in full in each case:

The product of the extremes equals etc.

If the product of two numbers equals the product of two  
numbers, either pair etc.

Alternation. (e) Composition.

Inversion. (f) Division.

Composition and division.

In a series of equal ratios, the sum of the antecedents  
the sum of the consequents etc.

Like powers or like roots of the terms of a proportion etc.

If  $x : m :: 13 : 7$ , write all the possible proportions that  
e derived from it. [See (5) above.]

Given  $rs = 161m$ ; write the eight proportions that may  
rived from it, and quote your authority.

(a) What theorem allows you to change any proportion  
an equation?

What theorem allows you to change any equation into a  
portion?

If  $xy = ry$ , what is the ratio of  $x$  to  $g$ ? of  $y$  to  $r$ ? of  $y$  to  $g$ ?

Find two numbers such that their sum, difference, and  
um of their squares are in the ratio 5 : 3 : 51. (Vale.)

ence: The chapter on Ratio and Proportion in any algebra.

Let  $\frac{a}{b} = r$ .  $\therefore a = br$ .

Also  $\frac{c}{d} = r$ .  $\therefore c = dr$ .

Substitute the value of  $a$  in the first ratio, and  $c$  in the second :

Then  $\frac{3a^3 + 5ab^2}{3a^3 - 5ab^2} = \frac{3b^3r^3 + 5b^2r}{3b^3r^3 - 5b^2r} = \frac{b^2r(3r^3 + 5)}{b^2r(3r^3 - 5)} = \frac{3r^3 + 5}{3r^3 - 5}$ .

Also  $\frac{3c^3 + 5cd^2}{3c^3 - 5cd^2} = \frac{3d^3r^3 + 5dr}{3d^3r^3 - 5dr} = \frac{d^2r(3r^3 + 5)}{d^2r(3r^3 - 5)} = \frac{3r^3 + 5}{3r^3 - 5}$ .

$\therefore \frac{3a^3 + 5ab^2}{3a^3 - 5ab^2} = \frac{3c^3 + 5cd^2}{3c^3 - 5cd^2}$ .

Axiom 1.

Or,  $3a^3 + 5ab^2 : 3a^3 - 5ab^2 = 3c^3 + 5cd^2 : 3c^3 - 5cd^2$ .

If  $a : b = c : d$ , prove:

1.  $a^2 + b^2 : a^2 = c^2 + d^2 : c^2$ .

2.  $a^2 + 3b^2 : a^2 - 3b^2 = c^2 + 3d^2 : c^2 - 3d^2$ .

3.  $a^2 + 2b^2 : 2b^2 = ac + 2bd : 2bd$ .

4.  $2a + 3c : 2a - 3c = 8b + 12d : 8b - 12d$ .

5.  $a^2 - ab + b^2 : \frac{a^3 - b^3}{a} = c^2 - cd + d^2 : \frac{c^3 - d^3}{c}$ .

6. The second of three numbers is a mean proportional between the other two. The third number exceeds the sum of the other two by 20; and the sum of the first and third exceeds three times the second by 4. Find the numbers.

7. Three numbers are proportional to 5, 7, and 9; and their sum is 14. Find the numbers. (*College Entrance Board*.)

8. A triangular field has the sides 15, 18, and 27 rods, respectively. Find the dimensions of a similar field having 4 times the area.

$$S = \frac{n}{2}(a + l),$$

$$S = \frac{n}{2}[2a + (n - 1)d].$$

- Find the sum of the first 50 odd numbers.
- In the series 2, 5, 8, ..., which term is 92?
- How many terms must be taken from the series 3, 5, 7, ... to make a total of 255?
- Insert 5 arithmetical means between 11 and 32.
- Insert 9 arithmetical means between  $7\frac{1}{2}$  and 30.
- Find  $x$ , if  $3 + 2x$ ,  $5 + 6x$ ,  $9 + 5x$  are in A. P.
- The 7th term of an arithmetical progression is 17, and the 13th term is 59. Find the 4th term.
- How can you turn an A. P. into an equation?
- Given  $a = -\frac{5}{3}$ ,  $n = 20$ ,  $S = -\frac{5}{3}$ , find  $d$  and  $l$ .
- Find the sum of the first  $n$  odd numbers.

An arithmetical progression consists of 21 terms. The middle term of the three terms in the middle is 129; the sum of the three terms is 237. Find the series. (Look up the short method for such problems.) (*Mass. Inst. of Technology*.)

B travels 3 miles the first day, 7 miles the second day, 11 miles the third day, etc. In how many days will B overtake A who started from the same point 8 days in advance and travels uniformly 15 miles a day?

**Exercise:** The chapter on Arithmetical Progression in any algebra.



$$\left\{ \text{II. } S = \frac{ar^n - a}{r - 1} \right.$$

$$\left\{ \text{IV. } S_{\infty} = \frac{a}{1 - r} \right.$$

2. How many terms must be taken from the series 9, 18, 36, ... to make a total of 567?

3. In the G. P. 2, 6, 18, ..., which term is 486?

4. Find  $x$ , if  $2x - 4$ ,  $5x - 7$ ,  $10x + 4$  are in geometrical progression.

5. How can you turn a G. P. into an equation?

6. Insert 4 geometrical means between 4 and 972.

7. Insert 6 geometrical means between  $\frac{1}{10}$  and 5120.

8. Given  $a = -2$ ,  $n = 5$ ,  $l = -32$ ; find  $r$  and  $S$ .

9. If the first term of a geometrical progression is 12 and the sum to infinity is 36, find the 4th term.

10. If the series  $3\frac{1}{2}$ ,  $2\frac{1}{2}$ , ... be an A. P., find the 97th term. If a G. P., find the sum to infinity.

11. The third term of a geometrical progression is 36; the 6th term is 972. Find the first and second terms.

12. Insert between 6 and 16 two numbers, such that the first three of the four shall be in arithmetical progression, and the last three in geometrical progression.

13. A rubber ball falls from a height of 40 inches and on each rebound rises 40% of the previous height. Find by formula how far it falls on its eighth descent. (Yale.)

**Reference:** The chapter on Geometrical Progression in any algebra.

$$(x + x^{-1})^n.$$

$$6. (x^2 - x + 2)^3.$$

$$\left(\frac{a-x}{x-a}\right)^n.$$

$$7. \left(2\sqrt[3]{b^3} + \frac{3\sqrt{y}}{b^3}\right)^4.$$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2$$

$$+ \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4}a^{n-4}b^4 + \dots$$

Now by observation that the formula for the

$$r\text{th term} = \frac{n(n-1)(n-2) \cdots (n-r+1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdots r} a^{n-r} b^r.$$

Indicate what the 97th term of  $(a+b)^n$  would be.

Using the expansion of  $(a+b)^n$  in (8), derive a formula for the  $r$ th term by observing how each term is made up, then realizing.

Using either the formula in (8) or (10), whichever you are familiar with, find:

$$1. \text{ The 4th term of } \left(a + \frac{1}{a}\right)^{30}.$$

$$2. \text{ The 8th term of } (1 + x\sqrt{y})^{13}.$$

$$3. \text{ The middle term of } (2a^{\frac{1}{2}} - y\sqrt[3]{a})^{10}.$$

$$4. \text{ The term not containing } x \text{ in } \left(x^3 - \frac{2}{x}\right)^{12}.$$

$$5. \text{ The term containing } x^{14} \text{ in } (x^2 - a)^{16}.$$

Reference: The chapter on 'The Binomial Theorem' in any algebra.

3. Solve  $2\sqrt{2x+2} + \sqrt{2x+1} = \frac{12x+1}{\sqrt{8x+8}}$ . (Yale.)

4. Solve the equation  $V = \frac{H}{3}(B+x+\sqrt{Bx})$  for  $x$ , taking  $H=6$ ,  $B=8$ , and  $V=28$ ; and verify your result. (Harvard.)

5. Solve  $\begin{cases} x:y=2:3, \\ x^2+y^2=5(x+y)+2. \end{cases}$

6. Solve  $2x^2-4x+3\sqrt{x^2-2x+6}=15$ . (Coll. Ent. Board.)

7. Find all values of  $x$  and  $y$  which satisfy the equations:

$\begin{cases} \sqrt{x} + \sqrt{y} = 4, \\ \frac{1}{\sqrt{x+1}-\sqrt{x}} - \frac{1}{\sqrt{x+1}+\sqrt{x}} = y. \end{cases}$  (Mass. Inst. of Technology.)

8. If  $\alpha$  and  $\beta$  represent the roots of  $px^2+qx+r=0$ , find  $\alpha+\beta$ ,  $\alpha-\beta$ , and  $\alpha\beta$  in terms of  $p$ ,  $q$ , and  $r$ . (Princeton.)

9. Form the equation whose roots are  $2+\sqrt{-3}$  and  $2-\sqrt{-3}$ .

10. Determine, without solving, the character of the roots of  $9x^2-24x+16=0$ . (College Entrance Board.)

11. If  $a:b=c:d$ , prove that  $a+b:c+d=\sqrt{a^2+b^2}:\sqrt{c^2+d^2}$ . (College Entrance Board.)

12. Given  $a:b=c:d$ . Prove that  $a^2+b^2:\frac{a^2}{a+b}=\frac{c^2}{c+d}:\frac{c^2}{c+d}$ . (Sheffield.)

13. The 9th term of an arithmetical progression is  $\frac{1}{6}$ ; the 16th term is  $\frac{1}{3}$ . Find the first term. (Regents.)

the sum of the first two is 1, and the sum of the last two is 19.

What number added to 2, 20, 9, 34, will make the terms proportional?

Find the middle term of  $\left(3a^6 + \frac{b^3}{2}\right)^8$ .

Solve  $\frac{x+1}{3x+2} = \frac{2x-3}{3x-2} - 1 - \frac{36}{4-9x^2}$ . (*Princeton.*)

A strip of carpet one half inch thick and  $29\frac{1}{7}$  feet long is rolled on a roller four inches in diameter. Find how many turns there will be, remembering that each turn increases the diameter by one inch, and that the circumference of a circle is (approximately)  $\frac{22}{7}$  times the diameter. (*Harvard.*)

The sum of the first three terms of a geometrical progression is 21, and the sum of their squares is 189. What is the fourth term? (*Yale.*)

Find the geometrical progression whose sum to infinity is 4 and whose second term is  $\frac{1}{2}$ .

Solve  $4x + 4\sqrt{3}x^2 - 7x + 3 = 3x^2 - 3x + 6$ .

Solve  $\begin{cases} 2x^2 + 3xy - 5y^2 = 4, \\ 2xy + 3y^2 = -3. \end{cases}$

Two hundred stones are placed on the ground 3 feet apart, the first being 3 feet from a basket. If the basket and the stones are in a straight line, how far does a person walk who starts from the basket and brings the stones to the basket one by one?

$$1. \begin{cases} x^2 + y^2 = 25, \\ x + y = 1. \end{cases}$$

$$2. x^2 - 3x - 18 = 0.$$

$$3. x^2 + 3x - 10 = 0.$$

Determine the value of  $m$  for which the roots of the equation will be equal: (Hint: See page 40. To have the roots equal,  $b^2 - 4ac$  must equal 0.)

$$4. 2x^2 - mx + 12\frac{1}{2} = 0.$$

$$5. (m-1)x^2 + mx + 2m - 3 = 0.$$

6. If  $2a + 3b$  is a root of  $x^2 - 6bx - 4a^2 + 9b^2 = 0$ , find the other root without solving the equation.

(Univ. of Penn.)

7. How many times does a common clock strike in 12 hours?

8. Find the sum to infinity of  $\frac{2}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \dots$

$$9. \text{Solve } \left(\frac{x}{2} + \frac{6}{x}\right)^2 - 6\left(\frac{x}{2} + \frac{6}{x}\right) + 8 = 0.$$

10. Find the value of the recurring decimal 2.214214 ...

11. A man purchases a \$500 piano by paying monthly installments of \$10 and interest on the debt. If the yearly rate is 6%, what is the total amount of interest?

12. The arithmetical mean between two numbers is  $42\frac{1}{2}$ , and their geometrical mean is 12. Find the numbers.

(College Entrance Exam. Board.)

13. If the middle term of  $\left(3x - \frac{1}{2\sqrt{x}}\right)^4$  is equal to the fourth term of  $\left(2\sqrt{x} + \frac{1}{2x}\right)^7$ , find the value of  $x$ . (M. I. T.)

miles an hour. How great is the distance? (Cornell.)

A man can walk  $2\frac{1}{2}$  miles an hour up hill and  $3\frac{1}{2}$  miles an hour down hill. He walks 56 miles in 20 hours on a road no of which is level. How much of it is up hill? (Yale.)

A physician having 100 cubic centimeters of a 6 % solution of a certain medicine wishes to dilute it to a  $3\frac{1}{2}$  % solution. How much water must he add? (A 6 % solution contains 6 % medicine and 94 % of water.) (Case.)

A clerk earned \$504 in a certain number of months. His salary was increased 25 %, and he then earned \$450 in two months less time than it had previously taken him to earn \$504. What was his original salary per month?

(College Entrance Board.)

A person who possesses \$15,000 employs a part of the money in building a house. He invests one third of the money in bonds which remain at 6 %, and the other two thirds at 9 %, and these investments he obtains an annual income of \$500. What was the cost of the house? (M. I. T.)

Two travelers have together 400 pounds of baggage. One pays \$1.20 and the other \$1.80 for excess above the weight allowed free. If all had belonged to one person, he would have to pay \$4.50. How much baggage is allowed to go free? (Yale.)

A man who can row  $4\frac{1}{2}$  miles an hour in still water rows down stream and returns. The rate of the current is  $2\frac{1}{2}$  miles an hour, and the time required for the trip is 13 hours. How long does he require to return?

these 1000 other shelves or men boards in such a way that the book space will diminish one inch for each shelf from the bottom to the top. What will be the several spaces between the shelves?

2. A quantity of water, sufficient to fill three jars of different sizes, will fill the smallest jar 4 times, or the largest jar twice with 4 gallons to spare, or the second jar three times with 2 gallons to spare. What is the capacity of each jar? (*Case.*)

3. A policeman is chasing a pickpocket. When the policeman is 80 yards behind him, the pickpocket turns up an alley; but coming to the end, he finds there is no outlet, turns back, and is caught just as he comes out of the alley. If he had discovered that the alley had no outlet when he had run halfway up and had then turned back, the policeman would have had to pursue the thief 120 yards beyond the alley before catching him. How long is the alley? (*Harvard.*)

4. A and B together can do a piece of work in 14 days. After they have worked 6 days on it, they are joined by C who works twice as fast as A. The three finish the work in 4 days. How long would it take each man alone to do it? (*Columbia.*)

5. In a certain mill some of the workmen receive \$1.50 a day, others more. The total paid in wages each day is \$350. An assessment made by a labor union to raise \$200 requires \$1.00 from each man receiving \$1.50 a day, and half of one day's pay from every man receiving more. How many men receive \$1.50 a day? (*Harvard.*)

Two automobiles travel toward each other over a distance of 60 miles. A leaves at 9 A.M., 1 hour before B starts to follow him, and they meet at 12:00 M. If each had started at 9 A.M., they would have met at 12:00 M. also. Find the rate at which each traveled.  
(M. I. T.)

### Quadratic Equations

Telegraph poles are set at equal distances apart. In order to have two less to the mile, it will be necessary to set the poles 20 feet farther apart. Find how far apart they are now.  
(Yale.)

The distance  $S$  that a body falls from rest in  $t$  seconds is given by the formula  $S = 16t^2$ . A man drops a stone into a well and hears the splash after 3 seconds. If the velocity of sound in air is 1086 feet a second, what is the depth of the well?  
(Yale.)

It requires 2000 square tiles of a certain size to pave a hall. If the hall is paved with 3125 square tiles whose dimensions are one inch less, the area of the hall is the same. How many solutions has the equation of this problem? How many has the problem itself? Explain the apparent discrepancy.  
(Cornell.)

A rectangular tract of land, 800 feet long by 600 feet wide, is divided into four rectangular blocks by two streets of equal width running through it at right angles. Find the width of the streets, if together they cover an area of 77,500 square feet.  
(M. I. T.)



(b) Draw the graph of the equation  $y = 100x - 16x^2$ .

(College Entrance Board.)

6. Two launches race over a course of 12 miles. The first steams  $7\frac{1}{2}$  miles an hour. The other has a start of 10 minutes, runs over the first half of the course with a certain speed, but increases its speed over the second half of the course by 2 miles per hour, winning the race by a minute. What is the speed of the second launch? Explain the meaning of the negative answer.

(Sheffield Scientific School.)

7. The circumference of a rear wheel of a certain wagon is 3 feet more than the circumference of a front wheel. The rear wheel performs 100 fewer revolutions than the front wheel in traveling a distance of 6000 feet. How large are the wheels?

(Harvard.)

8. A man starts from home to catch a train, walking at the rate of 1 yard in 1 second, and arrives 2 minutes late. If he had walked at the rate of 4 yards in 3 seconds, he would have arrived  $2\frac{1}{2}$  minutes early. Find the distance from his home to the station.

(College Entrance Board.)

### Simultaneous Quadratics

1. Two cubical coal bins together hold 280 cubic feet of coal, and the sum of their lengths is 10 feet. Find the length of each bin.

2. The sum of the radii of two circles is 25 inches, and the difference of their areas is  $125\pi$  square inches. Find the radii.

cube. (b) Find the distance from upper left-hand corner  
wor right-hand corner in either cube.

A and B run a mile. In the first heat A gives B a start  
0 yards and beats him by 30 seconds. In the second heat  
gives B a start of 32 seconds and beats him by  $9\frac{5}{11}$  yards.  
the rate at which each runs. (Sheffield.)

After street improvement it is found that a certain corner  
angular lot has lost  $\frac{1}{10}$  of its length and  $\frac{1}{10}$  of its width.  
perimeter has been decreased by 28 feet, and the new area  
24 square feet. Find the reduced dimensions of the lot.  
(College Entrance Board.)

A man spends \$539 for sheep. He keeps  $\frac{1}{4}$  of the flock  
he buys, and sells the remainder at an advance of \$2  
head, gaining \$28 by the transaction. How many sheep  
he buy, and what was the cost of each? (Yale.)

A boat's crew, rowing at half their usual speed, row 3  
s downstream and back again in 2 hours and 40 minutes.  
full speed they can go over the same course in 1 hour and  
minutes. Find the rate of the crew, and the rate of the cur-  
rent in miles per hour. (College Entrance Board.)

Find the sides of a rectangle whose area is unchanged if  
length is increased by 4 feet and its breadth decreased by  
4 feet, but which loses one third of its area if the length is  
increased by 16 feet and the breadth decreased by 10 feet.  
(M. I. T.)

1. If  $a = 4$ ,  $b = -3$ ,  $c = 2$ , and  $d = -4$ , find the value of:

(a)  $ab^3 - 3cd^2 + 2(3a - b)(c - 2d)$ .

(b)  $2a^3 - 3b^4 + (4c^2 + d^3)(4c^2 + d^3)$ .

2. Reduce to a mixed number:

$$\frac{3a^4 - 4a^3 - 10a^2 + 41a - 28}{a^2 - 3a + 4}.$$

Simplify:

3.  $\frac{a+2}{a^2+3a-40} - \frac{b-2}{ab-5b+3a-15}$ .

4.  $\left(1 - \frac{2-3b-2c}{a+2}\right) + \frac{a^2-4c^2+9b^2+6ab}{2a^2+a-6}$ .

5. A's age 10 years hence will be 4 times what B's age was 11 years ago, and the amount that A's age exceeds B's age is one third of the sum of their ages 8 years ago. Find their present ages.

6. Draw the lines represented by the equations

$$3x - 2y = 13 \text{ and } 2x + 5y = -4,$$

and find by algebra the coördinates of the point where they intersect.

7. Solve the equations  $\begin{cases} bx - ay = b^2 - ab, \\ y - b = 2(x - 2a). \end{cases}$

8. Solve  $(2x+1)(3x-2) - (5x-7)(x-2) = 41$ .

Solve by factoring:  $x^2 + 30x = 11x^2$ .

Show that  $1 - \left( \frac{a^2 + b^2 - c^2}{2ab} \right)^2$

$$= (a + b + c)(a + b - c)(a - b + c)(b + c - a) + 4a^2b^2.$$

How many pairs of numbers will satisfy simultaneously the equations

$$\begin{cases} 3x + 2y = 7, \\ x + y = 3? \end{cases}$$

How can you be sure by means of a graph that your answer is correct?  
What is meant by eliminating  $x$  in the above equations by  
substitution? by comparison? by subtraction?

Find the square root of 223,728.

Simplify: (a)  $\sqrt{\frac{1}{3}} + \sqrt{12} - \sqrt{\frac{1}{3}}$ .

$$(b) (-\sqrt{3} - 3\sqrt{3} - 4)^4.$$

Solve the equation

$$.03x^2 - 2.23x + 1.1075 = 0.$$

How far must a boy run in a potato race if there are  $n$   
pegs in a straight line at a distance  $d$  feet apart, the first  
peg at a distance  $a$  feet from the basket?

# ELEMENTARY ALGEBRA COMPLETE

TIME: THREE HOURS

Six questions are required; two from Group *A*, two from Group *B*, and both questions of Group *C*. No extra credit will be given for more than six questions.

## *Group A*

1. (a) Resolve the following into their prime factors:

(1)  $(x^2 - y^2)^2 - y^4$ .

(2)  $10x^2 - 7x - 6$ .

- (b) Find the H. C. F. and the L. C. M. of

$$x^3 - 3x^2 + x - 3,$$

$$x^3 - 3x^2 - x + 3.$$

2. (a) Simplify

$$\frac{\frac{x}{y} + \frac{y}{x} - 2}{\frac{1}{x} + \frac{1}{y}} + \frac{\frac{x}{y} + \frac{y}{x} + 2}{\frac{1}{x} - \frac{1}{y}}.$$

(b) If  $x : y = (x - z)^2 : (y - z)^2$ , prove that  $z$  is a mean proportional between  $x$  and  $y$ .

3. A crew can row 10 miles in 50 minutes downstream, and 12 miles in an hour and a half upstream. Find the rate of the current and of the crew in still water.

have equal roots.

Solve the equations

$$x^2 - xy + y^2 = 7,$$

$$2x - 3y = 0.$$

Plot the following two equations, and find from the graph the approximate values of their common solutions:

$$x^2 + y^2 = 25,$$

$$4x^2 + 9y^2 = 144.$$

Two integers are in the ratio 4 : 5. Increase each by 15, the difference of their squares is 999. What are the integers?

A man has \$539 to spend for sheep. He wishes to keep the flock that he buys, but to sell the remainder at a price of \$2 per head. This he does and gains \$28. How many sheep did he buy, and at what price each?

### Group C

(a) Find the seventh term of  $\left(a + \frac{1}{a}\right)^n$ .

Derive the formula for the sum of  $n$  terms of an arithmetic progression.

A ball falling from a height of 60 feet rebounds after each fall one third of its last descent. What distance has it traveled over when it strikes the ground for the eighth time?

1. Find the H.C.F.:

$$\begin{aligned}x^4 - y^4, \\ x^3 - xy^2 + x^2y - y^3, \\ x^4 + 2x^2y^2 - 3y^4.\end{aligned}$$

2. Solve the following set of equations:

$$\begin{aligned}x + y &= -1, \\ x + 3y + 2z &= -4, \\ x - y + 4z &= 5.\end{aligned}$$

3. Expand and simplify:

$$\left(2x^3 - \frac{1}{x}\right)^7.$$

4. An automobile goes 80 miles and back in 9 hours. The rate of speed returning was 4 miles per hour faster than the rate going. Find the rate each way.

5. Simplify:

$$\frac{\left(\frac{x+1}{x-1}\right)^2 - 2 + \left(\frac{x-1}{x+1}\right)^2}{\left(\frac{x+1}{x-1}\right)^2 - \left(\frac{x-1}{x+1}\right)^2}.$$

6. Solve for  $x$ :

$$\frac{2x+3}{x-1} - 6 = \frac{5}{x^2+2x-3}.$$

7. A, B, and C, all working together, can do a piece of work in  $2\frac{2}{3}$  days. A works twice as fast as C, and A and C together could do the work in 4 days. How long would it take each one of the three to do the work alone?

$$\begin{aligned}x + y &= -1, \\2z + 5w &= 1,\end{aligned}$$

$$\begin{aligned}x + 3y + 2z &= -4, \\x - y + 4z + 4w &= 5.\end{aligned}$$

Simplify: (a)  $\sqrt{6} - \sqrt{20}$ . (b)  $\frac{1 + \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1} + x^2}$ .

Find, and simplify, the 23d term in the expansion of

$$\left(\frac{2x^2}{3} - \frac{3}{4}\right)^{28}.$$

The weight of an object varies directly as its distance from the center of the earth when it is below the earth's surface and inversely as the square of its distance from the center when it is above the surface. If an object weighs 10 pounds at the surface, how far above, and how far below the surface will it weigh 9 pounds? (The radius of the earth may be taken as 4000 miles.)

Solve the following pair of equations for  $x$  and  $y$ :

$$\begin{aligned}x^2 + y^2 &= 1, \\x &= (1 + \sqrt{2})y + 2.\end{aligned}$$

Find the value of  $\frac{1 + 8^{-x}}{(8x)^{\frac{1}{4}} + 10^{-x}}$ , when  $x = 2$ .

From a square of pasteboard, 12 inches on a side, squares are cut, and the sides are turned up to form a rectangular box. If the squares cut out from the corners had been larger on a side, the volume of the box would have increased 28 cubic inches. What is the size of the square cut out? (See the figure on the blackboard.)



Arrange your work neatly and clearly, beginning each question on a separate page.

1. Simplify the following expression:

$$\frac{\frac{1}{a} + \frac{1}{b+c}}{\frac{1}{a} - \frac{1}{b+c}} \left[ 1 + \frac{b^2 + c^2 - a^2}{2bc} \right].$$

2. (a) Write the middle term of the expansion of  $(a-b)^{14}$  by the binomial theorem.

- (b) Find the value of  $a^2b^2$ , if

$$a = x^{\frac{2}{3}}y^{-\frac{1}{3}} \quad \text{and} \quad b = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}},$$

and reduce the result to a form having only positive exponents.

3. Find correct to three significant figures the negative root of the equation

$$1 - \frac{2}{x+1} + \frac{4x}{(x+1)^2} = 0.$$

4. Prove the rule for finding the sum of  $n$  terms of a geometrical progression of which the first term is  $a$  and the constant ratio is  $r$ .

Find the sum of 8 terms of the progression

$$5 + 3\frac{1}{2} + 2\frac{3}{8} + \dots$$

5. A goldsmith has two alloys of gold, the first being  $\frac{3}{4}$  pure gold, the second  $\frac{5}{12}$  pure gold. How much of each must he take to produce 100 ounces of an alloy which shall be  $\frac{2}{3}$  pure gold?

4. Solve the simultaneous equations

$$x + y = a + b,$$

$$\frac{y + b}{x + a} = \frac{a}{b},$$

and verify your results.

5. Solve the equation  $x^2 - 1.6x - 0.23 = 0$ , obtaining the values of the roots correct to three significant figures.

6. Write out the first four terms of  $(a - b)^7$ .

Find the fourth term of this expansion when

$$a = \sqrt[3]{x^{-1}y^4},$$

$$b = \sqrt[3]{9xy^{-4}},$$

expressing the result in terms of a single radical, and without fractional or negative exponents.

7. Reduce the following expression to a polynomial in  $a$  and  $b$ :

$$\frac{6a^3 + 7ab^2 + 12b^3}{3a^2 - 5ab - 4b^2} - \frac{1}{\frac{3}{19b} - \frac{5a + 4b}{19a^2}}$$

8. The cost of publishing a book consists of two main items: first, the fixed expense of setting up the type; and, second, the printing expenses of presswork, binding, etc., which may be assumed to be proportional to the number of copies. A certain book costs 35 cents a copy if 1000 copies are published at one time, but only 19 cents a copy if 5000 copies are published at one time. Find (a) the cost of setting up the type for the book, and (b) the cost of presswork, binding, etc., per thousand copies.

1. Find the highest common factor and the lowest common multiple of the three expressions

$$a^4 - b^4; \quad a^3 + b^3; \quad a^3 + 2a^2b + 2ab^2 + b^3.$$

2. Solve the quadratic equation

$$x^2 - 1.6x + 0.3 = 0,$$

computing the value of the larger root correct to three significant figures

3. In the expression

$$x^2 - 2xy + y^2 - 4\sqrt{2}(x + y) + 8,$$

substitute for  $x$  and  $y$  the values

$$x = \frac{u + v + 1}{\sqrt{2}}, \quad y = \frac{u - v + 1}{\sqrt{2}},$$

and reduce the resulting expression to its simplest form.

4. State and prove the formula for the sum of the first  $n$  terms of a geometric progression in which  $a$  is the first term and  $r$  the constant ratio.

5. A state legislature is to elect a United States senator, a majority of all the votes cast, being necessary for a choice. There are three candidates, A, B, and C, and 100 members vote. On the first ballot A has the largest number of votes, receiving 9 more votes than his nearest competitor, B; but he fails of the necessary majority. On the second ballot C's name is withdrawn, and all the members who voted for C now vote for B, whereupon B is elected by a majority of 2. How many votes were cast for each candidate on the first ballot?

1. Factor the expressions:

$$x^3 + x^2 - 2x.$$

$$x^3 + x^2 - 4x - 4.$$

2. Simplify the expression:

$$\left(1 - \frac{b^2}{a^2}\right)\left(1 - \frac{ab - b^2}{a^2}\right) \frac{a^4}{a^3 + b^3} \cdot \frac{a - b}{a^2 + b^2}.$$

3. Find the value of  $x + \sqrt{1 + x^2}$ , when  $x = \frac{1}{2} \left( \sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}} \right)$ .

4. Solve the equations:

$$\frac{7x + 6}{11} + y - 16 = \frac{5x - 13}{2} - \frac{8y - x}{5},$$

$$3(3x + 4) = 10y - 15.$$

5. Solve the equations:

$$\begin{aligned} A &+ C = 2, \\ -A + B + C + D &= 1, \\ 2A - B + 2C + D &= 5, \\ B &+ D = 1. \end{aligned}$$

6. Two squares are formed with a combined perimeter of 8 inches. One square contains 4 square inches more than the other. Find the area of each.

7. A man walked to a railway station at the rate of 4 miles an hour and traveled by train at the rate of 30 miles an hour, reaching his destination in 20 hours. If he had walked 3 miles an hour and ridden 35 miles an hour, he would have made the journey in 18 hours. Required the total distance traveled.

1. How many terms must be taken in the series 2, 5, 8, 11, ... so that the sum shall be 345?

2. Prove the formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  for solving the quadratic equation  $ax^2 + bx + c = 0$ .

3. Find all values of  $a$  for which  $\sqrt{a}$  is a root of  $x^2 + x + 20 = 2a$ , and check your results.

4. Solve  $\begin{cases} x^2 + 3y^2 = 10, \\ x - y = 2, \end{cases}$  and sketch the graphs.

5. The sum of two numbers  $x$  and  $y$  is 5, and the sum of the two middle terms in the expansion of  $(x + y)^3$  is equal to the sum of the first and last terms. Find the numbers.

6. Solve  $x^4 - 2x^3 + 3x^2 - 2x + 1 = 0$ .

(Hint: Divide by  $x^2$  and substitute  $x + \frac{1}{x} = z$ .)

7. In anticipation of a holiday a merchant makes an outlay of \$50, which will be a total loss in case of rain, but which will bring him a clear profit of \$150 above the outlay if the day is pleasant. To insure against loss he takes out an insurance policy against rain for a certain sum of money for which he has to pay a certain percentage. He then finds that whether the day be rainy or pleasant he will make \$80 clear. What is the amount of the policy, and what rate did the company charge him?

$$1. \text{ Simplify } \left(m + \frac{1}{m}\right)^2 + \left(n + \frac{1}{n}\right)^2 + \left(mn + \frac{1}{mn}\right)^2 \\ - \left(m + \frac{1}{m}\right)\left(n + \frac{1}{n}\right)\left(mn + \frac{1}{mn}\right).$$

2. Find the prime factors of

$$(a) (x - x^2)^3 + (x^2 - 1)^3 + (1 - x)^3.$$

$$(b) (2x + a - b)^4 - (x - a + b)^4.$$

$$3. (a) \text{ Simplify } \left(\frac{x^2}{x^r}\right)^{r+1} \left(\frac{x^r}{x^p}\right)^{r+1} \left(\frac{x^p}{x^2}\right)^{r+1}.$$

$$(b) \text{ Show that } \frac{\sqrt[n]{n+1} \sqrt[n]{x}}{\sqrt[n+1]{n+2} \sqrt[n]{x}} = \frac{\sqrt[n]{x} \cdot \sqrt[n+2]{x}}{\sqrt[n+1]{x^2}}.$$

4. Define *homogeneous terms*.

For what value of  $n$  is  $x^a y^{b-\frac{n}{2}} + x^{a+1} y^{2n-1}$  a homogeneous monomial?

5. Extract the square root of

$$x(x - \sqrt{2})(x - \sqrt{8})(x - \sqrt{18}) + 4.$$

6. Two vessels contain each a mixture of wine and water. In the first vessel the quantity of wine is to the quantity of water as 1 : 3, and in the second as 3 : 5. What quantity must be taken from each, so as to form a third mixture which shall contain 5 gallons of wine and 9 gallons of water?

7. Find a quantity such that by adding it to each of the quantities  $a, b, c, d$ , we obtain four quantities in proportion.

8. What values must be given to  $a$  and  $b$ , so that

$$\frac{a + 2b + 17}{2}, \frac{2a - 3b + 25}{3}, \text{ and } 4 - 5a - 13b \text{ may be equal?}$$

Factor the following expressions:

1)  $a^{\frac{3}{4}} - b^{\frac{3}{4}}$ .

2)  $x^2y^2z^2 - x^2z - y^2z + 1$ .

3)  $16(x+y)^4 - (2x-y)^4$ .

4) Simplify

$$(a^2 + b^2) \left\{ \frac{\frac{b^4}{b^2 - a^2} - a^2}{\frac{a}{a+b} + \frac{b}{a-b}} \right\}.$$

5) Extract the square root of  $x^4 - 2x^3 + 5x^2 - 4x + 4$ .

6) Solve the following equations:

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = 5, \\ \frac{1}{x^2} + \frac{1}{y^2} = 13. \end{cases}$$

$$x^2 - 5x + 2 = 0.$$

$$\sqrt{27x+1} = 2 - 3\sqrt{3x}.$$

7) Simplify:

$$7\sqrt[3]{54} + \sqrt[3]{256} + \sqrt[3]{-\frac{432}{250}}.$$

$$\frac{1}{(a-b)(b-c)} + \frac{1}{(c-a)(b-a)}.$$

8) Find

$$\sqrt{19 - 8\sqrt{3}}.$$

$$(2x - 5y = 4.$$

6. (a) Derive the formula for the solution of

$$ax^2 + bx + c = 0.$$

(b) Determine the value of  $m$  for which the roots of  $x^2 + 4x + m = 0$  are (i) equal, (ii) real, (iii) imaginary.

(c) Form the quadratic equation whose roots are

$$2 + \sqrt{3} \text{ and } 2 - \sqrt{3}.$$

7. A page is to have a margin of 1 inch, and is to contain 5 square inches of printing. How large must the page be, if the length is to exceed the width by 2 inches?

8. (a) In an arithmetical progression the sum of the first six terms is 261, and the sum of the first nine terms is 297. Find the common difference.

(b) Three numbers whose sum is 27 are in arithmetical progression. If 1 is added to the first, 3 to the second, and 1 to the third, the sums will be in geometrical progression. Find the numbers.

(c) Derive the formula for the sum of  $n$  terms of a geometrical progression.

9. (a) Expand and simplify  $(2a^2 - 3x^3)^7$ .

(b) For what value of  $x$  will the ratio  $7 + x : 12 + x$  be equal to the ratio  $5 : 6$ ?



$$1. \text{ Simplify: } \left( \frac{a+b}{a-x} - \frac{a-b}{a+x} \right) \div \frac{a^2-b^2}{x^2}$$

2. Find the H. C. F. and L. C. M. of

$$10ab^2(x^2-2ax), 15a^3b(x^2-ax-2a^2), 25b^3(x^2-a^2)^2.$$

3. A grocer buys eggs at 4 for 7¢. He sells  $\frac{1}{4}$  of them at 5 for 12¢, and the rest at 6 for 11¢, making 27¢ by the transaction. How many eggs does he buy?

$$4. \text{ Solve for } t: \frac{t+4a+b}{t+a+b} - \frac{4t-a-2b}{t+a-b} = -3.$$

$$5. \text{ Find the square root of } a^3 - \frac{3}{2}a^{\frac{3}{2}} - \frac{3}{2}a^{\frac{1}{2}} + \frac{1}{8}a+1.$$

6. (a) For what values of  $m$  will the roots of  $2x^2+3mx=-2$  be equal?

(b) If  $2a+3b$  is a root of  $x^2-6bx-4a^2+9b^2=0$ , find the other root without solving the equation.

$$7. (a) \text{ Solve for } x: \sqrt{2}x-3a+\sqrt{3}x-2a=3\sqrt{a}.$$

$$(b) \text{ Solve for } m: 1 - \frac{1}{2-m} = \frac{1}{m+2} + \frac{m-6}{4-m^2}.$$

$$8. \text{ Solve the system: } x^2+2y^2=17; xy-y^2=2.$$

9. Two boats leave simultaneously opposite shores of a river  $2\frac{1}{2}$  mi. wide and pass each other in 15 min. The faster boat completes the trip  $6\frac{3}{4}$  min. before the other reaches the opposite shore. Find the rates of the boats in miles per hour.

10. Write the sixth term of  $\left(2\sqrt[3]{y^2} - \frac{\sqrt{y}}{x}\right)^9$  without writing the preceding terms.

11. The sum of the 2d and 20th terms of an A. P. is 10, and their product is  $23\frac{1}{3}$ . What is the sum of sixteen terms?

unit from Algebra A questions 4, 5, and 6, and from Algebra B questions 1 (a), 3, and 4.

Simplify

$$\frac{a^3 + a^2b + ab^2}{-3ab - 4b^2} \div \left\{ \frac{a^2 + 6ab - 7b^2}{a^2 + 8ab - 9b^2} \cdot \frac{a^3 - b^3}{a^2 - 7ab + 12b^2} \right\}.$$

(a) Divide  $a^{\frac{5}{2}} + ab^{\frac{3}{2}} + b^{\frac{5}{2}} - 2a^{\frac{1}{2}}b^2 - a^{\frac{3}{2}}b$  by  $a^{\frac{3}{2}} - b^{\frac{3}{2}} + a^{\frac{1}{2}}b - ab^{\frac{1}{2}}$ .

(b) Simplify  $\frac{1}{x^{-1} + y^{-1}} \cdot (x^{\frac{1}{2}}\sqrt{y})^3 + 1$ .

Factor: (a)  $(x^2 - 3x)^2 - (2x - 6)^2$ .  
(b)  $a^2 + ac - 4b^2 - 2bc$ .

Solve  $\frac{1}{x+1} - \frac{1}{x-1} - \frac{1}{x-3} + \frac{1}{x-5} = 0$ .

Solve for  $x$  and  $y$ :  $mx + ax = my - by$ ,  
 $x - y = a + b$ .

The road from A to B is uphill for 5 mi., level for 4 mi., and downhill for 6 mi. A man walks from B to A in 4 hr.; he walks halfway from A to B and back again to A in 55 min.; and later he walks from A to B in 3 hr. and 15 min. What are his rates of walking uphill, downhill, and level, if these do not vary?

## ALGEBRA B

Solve: (a)  $\frac{x+1}{x-2} + \frac{2x+1}{x+1} + \frac{3x+3}{1-x} = 0$ .

(b)  $\sqrt{2x+7} + \sqrt{3x-18} - \sqrt{7x+1} = 0$ .

(c)  $\frac{6}{x^2 + 2x} = 5 - 2x - x^2$ .

3. A man arranges to pay a debt of \$3600 in 40 monthly payments which form an A. P. After paying 30 of them he still owes  $\frac{1}{3}$  of his debt. What was his first payment?

4. If 4 quantities are in proportion and the second is a mean proportional between the third and fourth, prove that the third will be a mean prop. between the first and second.

5. In the expansion of  $\left(2x + \frac{1}{3x}\right)^6$  the ratio of the fourth term to the fifth is 2 : 1. Find  $x$ .

6. Two men A and B can together do a piece of work in 12 days; B would need 10 days more than A to do the whole work. How many days would it take A alone to do the work?

### ALGEBRA TO QUADRATICS

1. Simplify  $(ab^{-2}c^2)^{\frac{1}{2}} \cdot (a^2b^2c^{-3})^{\frac{1}{2}} + \sqrt[3]{\frac{a^6}{b}}$ .

2. Simplify  $\frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)}$ .

3. Factor (a)  $x^4 - 10x^2 + 9$ . (b)  $x^2 + 2xy - a^2 - 2ay$ .  
(c)  $(a+b)^2 + (a+c)^2 - (c+d)^2 - (b+d)^2$ .

4. Find H. C. F. of  $x^4 - x^3 + 2x^2 + x + 3$  and  $(x+2)(x^3-1)$ .

5. Solve  $\frac{x}{x-2} + \frac{x-9}{x-7} = \frac{x+1}{x-1} + \frac{x-8}{x-6}$ .

6. The sum of three numbers is 54; if the first number be divided by the second, the quotient is 2 and the remainder 5; if the second number be divided by the third, the quotient is 3 and the remainder 2. What are the numbers?

1. Factor  $e^{2x} - 2 + e^{-2x}$ ,  $x^{12} - 8$ ,  $x^2 - x - y^2 - y$ ,  $18a^2x^2 - 4axy - 10y^2$ .

2. Solve  $\sqrt{7 + 4x} + 3\sqrt{2x^2 + 5x + 7} - 3 = 0$ .

3. The second term of a geometrical progression is  $3\sqrt{2}$ , and the fifth term is  $\frac{3}{16}$ . Find the first term and the ratio.

4. Solve the following equations and check your results by totting:

$$\begin{cases} x^2 + y^2 - xy = 7, \\ x + y = 4. \end{cases}$$

5. Solve 
$$\begin{cases} \frac{1}{x^3} + \frac{1}{y^3} = \frac{243}{8}, \\ \frac{1}{x} + \frac{1}{y} = \frac{9}{2}. \end{cases}$$

6. In an arithmetical progression  $d = -11$ ,  $n = 13$ ,  $s = 0$ . Find  $a$  and  $l$ .

7. Expand by the binomial theorem and simplify:

$$\left( \frac{2x}{y^3} - \frac{y^4}{x^6\sqrt{-6}} \right)^6.$$

8. The diagonal of a rectangle is 13 ft. long. If each side were longer by 2 ft., the area would be increased by 38 sq. ft. Find the lengths of the sides.

1. Find the H.C.F. of  $8x^3 - 27$ ,  $32x^3 - 243$ , and  $6x^3 - 9x^2 + 4x - 6$ .

2. Solve:

$$(a) (2x+5)^{-3} + 31(2x+5)^{-\frac{1}{2}} = 32.$$

$$(b) (x-1)^{\frac{1}{2}} + (3x+1)^{\frac{1}{2}} = 4.$$

3. A farmer sold a horse at \$75 for which he had paid  $x$  dollars. He realized  $x$  per cent profit by his sale. Find  $x$ .

4. Find the 13th term and the sum of 13 terms of the arithmetical progression

$$\frac{\sqrt{2}-1}{2}, \quad \frac{\sqrt{2}}{2}, \quad \frac{1}{2(\sqrt{2}-1)}, \quad \dots$$

5. The difference between two numbers is 48. Their arithmetical mean exceeds their geometrical mean by 18. Find the numbers.

6. Expand by the binomial theorem and simplify

$$\left(3a^{\frac{1}{2}} - \frac{a}{\sqrt{a-2}}\right)^6.$$

7. Solve:

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{3}{2}, \\ \frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{4}. \end{cases}$$

8. Solve the following equations and check the results by finding the intersections of the graphs of the two equations:

$$\begin{cases} x^2 = 4y, \\ x + 2y = 4. \end{cases}$$

$$\left(1 + \frac{2a}{3} - \frac{5a^2}{6}\right) \text{ and } \left(2 - \frac{3a}{4} + \frac{a^2}{3}\right).$$

2. Resolve into linear factors:

(a)  $4x^2 - 25$ ; (b)  $6x^2 - x - 12$ ; (c)  $a^2b^2 + 1 - a^2 - b^2$ ;

(d)  $y^3 + (x-3)y^2 - (3x-2)y + 2x$ .

3. Reduce to simplest form:

(a)  $\frac{z}{1 - \frac{1}{y}} + \frac{y}{1 - \frac{y}{x}} - \frac{x}{1 - \frac{x}{y}}$ . (b)  $\{-(x^3)^{\frac{1}{3}}\}^{\frac{1}{3}} \times (4y^{-3})^{\frac{1}{3}}$ .

4. (a) Divide  $x^{\frac{3}{2}} - x^{-\frac{3}{2}}$  by  $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ .

(b) Find correct to one place of decimals the value of

$$\frac{\sqrt{5} + \sqrt{7}}{2 - \sqrt{3}}.$$

5. (a) If  $\frac{a}{b} = \frac{c}{d}$ , show that  $\frac{a^2 + c^2}{b^2 + d^2} = \frac{ac}{bd}$ .

(b) Two numbers are in the ratio 3:4, and if 7 be subtracted from each the remainders are in the ratio 2:3. Find the numbers.

6. Solve the equations:

(a)  $\frac{x+1}{2} - \frac{3}{x} = \frac{x-5}{3} - \frac{x}{6}$ . (c)  $\begin{cases} x^2 - 2y^2 = 71, \\ x + y = 20. \end{cases}$

(b)  $11x^2 - 11\frac{1}{2} = 9x$ .

7. A field could be made into a square by diminishing the length by 10 feet and increasing the breadth by 5 feet, but its area would then be diminished by 210 square feet. Find the length and the breadth of the field.

1. Find two numbers whose ratio is 3 and such that two sevenths of the larger is 15 more than one half the smaller.

2. Determine the factors of the lowest common multiple of  $3x^4(x^3 - y^3)$ ,  $15(x^4 - 2x^2y^2 + y^4)$ , and  $10y(x^4 + x^2y^2 + y^4)$ .

3. Find to two decimal places the value of

$$\sqrt[2]{4a^{-\frac{1}{2}} + b^0 \sqrt{ab^{-1}}}, \text{ when } a = -.32 \text{ and } b = -.8.$$

4. Solve the equations:  $2x + 5y = 85,$   
 $2y + 5z = 103,$   
 $2z + 5x = 57.$

5. Solve any 3 of these equations:

$$(a) x^2 + 44 - 15x = 0. \quad (c) x^2 + 8x - \sqrt{4x^2 + 32x + 12} = 21.$$

$$(b) \frac{2}{x} - \frac{x}{5} = \frac{x}{20} - \frac{223}{30}. \quad (d) \frac{5}{x+1} + \frac{8}{x-2} = \frac{12}{40-2x}.$$

6. The sum of two numbers is 13, and the sum of their cubes is 910. Find the smaller number, correct to the second decimal place.

7. The sum of 9 terms of an arithmetical progression is 46; the sum of the first 5 terms is 25. Find the common difference.

8. Explain the terms, and prove that if four numbers are in proportion, they are in proportion by *alternation*, by *inversion*, and by *composition*. Find  $x$  when

$$\frac{3+x}{3-x} = \frac{40+x^3}{40-x^3}.$$

9. Find the value of  $x$  in each of these equations:

$$(a) 7x^{\frac{1}{2}} - 3x^{\frac{1}{2}} = 2. \quad (b) (x^2 + 2)^{\frac{3}{2}} + \frac{3}{\sqrt{x^2 + 2}} = 4x^2 + 8.$$

*Group I*

Resolve into prime factors: (a)  $6x^2 - 7x - 20$ ;  
 $(x^2 - 5x)^2 - 2(x^2 - 5x) - 24$ ; (c)  $a^4 + 4a^2 + 16$ .

Simplify  $\left(5 - \frac{a^2 - 19x^2}{a^2 - 4x^2}\right) + \left(3 - \frac{a - 5x}{a - 2x}\right)$ .

Solve  $\frac{2(x+7)}{x^2+3x-28} + \frac{2-x}{4-x} - \frac{x+3}{x+7} = 0$ .

*Group II*

Simplify  $\frac{\sqrt{2} + 2\sqrt{3}}{\sqrt{2} - \sqrt{12}}$ , and compute the value of the fraction to two decimal places.

Solve the simultaneous equations  $\begin{cases} x^{-1} + 2y^{-1} = 7, \\ 2x^{-1} - y^{-1} = \frac{5}{6}. \end{cases}$

*Group III*

Two numbers are in the ratio of  $c:d$ . If  $a$  be added to first and subtracted from the second, the results will be in ratio of 3:2. Find the numbers.

A dealer has two kinds of coffee, worth 30 and 40 cents pound. How many pounds of each must be taken to make mixture of 70 pounds, worth 36 cents per pound?

A, B, and C can do a piece of work in 30 hours. A can all as much again as B, and B two thirds as much again as C. How long would each require to do the work alone?



Only one question in Group I and one in Group II. Credit will be given for *five* questions only.

*Group I*

1. Solve  $\frac{x+a}{x+b} + \frac{x+b}{x+a} = \frac{5}{2}$ .
2. Solve the simultaneous equations 
$$\begin{cases} x^2y^2 + 28xy + 180 = 0, \\ 2x + y = 11. \end{cases}$$

Arrange the roots in corresponding pairs.

3. Solve  $3x^4 + 20x^2 + 32 = 0$ .

*Group II*

4. In going 7500 yd. a front wheel of a wagon makes 1000 more revolutions than a rear one. If the wheels were each 1 yd. greater in circumference, a front wheel would make 625 more revolutions than a rear one. Find the circumference of each.

5. Two cars of equal speed leave A and B, 20 mi. apart, at different times. Just as the cars pass each other an accident reduces the power and their speed is decreased 40 mi. per hour. One car makes the journey from A to B in 56 min., and the other from B to A in 72 min. What is their common speed?

*Group III*

6. Write in the simplest form the last three terms of the expansion of  $(1 + a^{\frac{1}{2}} - a^{\frac{1}{2}}x^{\frac{1}{2}})^n$ .

7. (a) Derive the formula for the sum of an A. P.

(b) Find the sum to infinity of the series  $1, -\frac{1}{2}, \frac{1}{4}, \dots$ . Also find the sum of the positive terms.